EXERCISE QUESTIONS

CHAPTER - 2 ELECTROSTATIC POTENTIAL AND CAPACITANCE

2.1 Two charges 5 × 10–8 C and –3 × 10–8 C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Ans - Two charges qA = 5 x 10-8 C and qB = -3x10-8 C

Distance between two charges, r = 16 cm = 0.16 m

Consider a point O on the line joining two charges where the electric potential is zero due to two charges.

$$i.e. \quad 9 \times 10^9 \frac{q_A}{x} + 9 \times 10^9 \frac{q_B}{r-x} = 0$$

$$i.e. \quad 9 \times 10^9 \left[\frac{5\times10^{-8}}{x} + \frac{(-3\times10^{-8})}{(0{\cdot}16-x)}\right] = 0$$

$$i.e. \quad \frac{5\times10^{-8}}{x} = \frac{3\times10^{-8}}{(0{\cdot}16-x)}$$

$$i.e. \quad 5(0{\cdot}16 - x) = 3x$$

$$i.e. \quad 0{\cdot}8 = 3x + 5x = 8x$$

$$\text{or} \quad x = 0{\cdot}1 \text{ m} = 10 \text{ cm.}$$

2.2 A regular hexagon of side 10 cm has a charge 5 µC at each of its vertices. Calculate the potential at the centre of the hexagon.

Ans -

It follows that the point O,

when joined to the two ends of a side of the hexagon forms an equilateral triangle

Electric potential at O due to one charge

Total potential at O is given by,

$$V_1 = \frac{q}{4\pi\varepsilon_0 r}$$

Here, q = amount of charge = 5×10^{-6} C

r = distance between charge and O = 0.1 m

ε_0 = absolute permittivity of free space

Since at the each of the hexagon, a charge of 5 µC (5×10^{-6} C) is placed, total electric potential at the point O due to the charges at the six corners,

$$\mathbf{V} = 6 \times \frac{q}{4\pi\varepsilon_0 r}$$

$$\Rightarrow \mathbf{V_1} = 6 \times \frac{1}{4\pi\varepsilon_0} \times \frac{5\times10^{-6}\ \text{C}}{0.1\ \text{m}}$$

$$\Rightarrow \mathbf{V_1} = 6 \times \frac{9\times10^{9}\ \text{N m}^2\ \text{C}^{-2}\times5\times10^{-6}\ \text{C}}{0.1\ \text{m}}$$

$$\Rightarrow V_1 = 2.7 \times 10^{6}\ \text{V}$$

2.3 Two charges 2 µC and –2 µC are placed at points A and B 6 cm apart.
(a) Identify an equipotential surface of the system.
(b) What is the direction of the electric field at every point on this surface?

Ans - (a) The equipotential surface for the given system of two charges will be a plane that runs perpendicular to the line AB that connects the two charges and passes through its midpoint O. The potential is 0 at every point on this plane.

(b) The electric field is normal to the equipotential surface and extends from point A to point B, or from positive charge to negative charge.

2.4 A spherical conductor of radius 12 cm has a charge of 1.6 × 10–7C distributed uniformly on its surface. What is the electric field
(a) inside the sphere
(b) just outside the sphere
(c) at a point 18 cm from the centre of the sphere?

Ans - (a) Because the charge is located on a conductor's surface, there is no electric field inside the conductor.

(b) The electric field is given by immediately outside the sphere.

(c)

$$E_1 = \frac{q}{4\pi\epsilon_0 d^2}$$

$$= \frac{9\times10^{9}\times1.6\times10^{-7}}{\left(18\times10^{-2}\right)^2}$$

$$= 4.4\times10^{4}\ \text{N/C}$$

2.5 A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10–12 F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?
Ans -

$$\text{Using } C' = \frac{\epsilon_0 \epsilon_r A}{d}, \text{ we get}$$

$$C' = \epsilon_r C = 12(8 \times 10^{-12})$$
$$= 96 \times 10^{-12} \text{ F} = 96 \text{ pF}.$$

2.6 Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Ans - (a) Capacitance of each of the three capacitors, C = 9 pF

Equivalent capacitance (C’) is given by

1C'=1C+1C+1C⇒1C'=19+19+19=13⇒C'=3 pF1C'=1C+1C+1C⇒1C'=19+19+19=13⇒C'=3 pF

Therefore, total capacitance of the combination is 3 pF3 pF.

(b) Supply voltage, V = 120 V

Potential difference (V') across each capacitor is equal to one-third of the supply voltage.

$$\therefore V' = \frac{V}{3} = \frac{120}{3} = 40 \text{ V}$$

Therefore, the potential difference across each capacitor is 40 V.

2.7 Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.
(a) What is the total capacitance of the combination?
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.
Ans - (a) Given, C1 = 2 pF

C2 = 3 pF

C3 = 4 pF , V = 100 volt

Total capacitance
= c1 + c2 + c3
= 2 + 3+ 4 = 9pF.

(b) Let q1 , q2 and q3 be that charges on the capacitor C1 , C2 and C3 respectively.

Using C = qv we get q = CV
∴ qx = C1V = 2 x 10-12 x 100
= 2 x 10-10 C = 200 pC
q2 = c2V
= 3 x 10-12 x 100
= 3 x 10-10 C = 300 pC
q3 = c3v
= 4 x 10-12 x 100
= 4 x 10-10 C = 400 pC

2.8 In a parallel plate capacitor with air between the plates, each plate has an area of 6 × 10–3 m2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Ans -

Given, $A = 6 \times 10^{-3}\ m^2$

$D = 3\ mm = 3 \times 10^{-3}\ m$

Capacitance of the capacitor,

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12}\ Fm^{-1} \times 6 \times 10^{-3} m^2}{3 \times 10^{-3}\ m}$$

$\Rightarrow C = 1.77 \times 10^{-11}\ F = 17.7\ pF$

When the capacitor is connected to a 100 V supply, the charge on the each plate of the capacitor,

$q = CV = 1.77 \times 10^{-11} \times 100 = 1.77 \times 10^{-9}\ C$

2.9 Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,
(a) while the voltage supply remained connected.
(b) after the supply was disconnected.

Ans - C' = kC

Where k = dielectric constant = 6×17.7pF = 106.2 pF

The potential difference across the two plates of the capacitor will remain equal to the supply voltage i.e. 100 V

The charge on the capacitor,

$$(a)\ (i)\ C' = \epsilon_r C = 6 \times 18 = 108 \text{ pF}$$

$$(ii)\ q = C'V = 108 \times 100$$

$$= 1{\cdot}08 \times 10^{-8} \text{ C}$$

$$(b)\ q \text{ remains } 1{\cdot}8 \times 10^{-9} \text{ C;}$$

$$\text{Capacitance} = 108 \text{ pF}$$

$$\therefore \quad V = \frac{q}{C} = \frac{1{\cdot}8 \times 10^{-9}}{108 \times 10^{-12}} = 16{\cdot}6 \text{ V.}$$

2.10 A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?
Ans -

Given, C = 12 pF = 12 x 10-12 F

V = 50 V

The electrostatic energy stored in the capacitor,

is given by the relation,

W = (½) CV2

= (½) × 12 × 10-12× (50)2

= 1.5 × 10-8 J

2.11 A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?
Ans - Given, C1 = 600 pF = 600 x 10-12 F

V1 = 200 V

Energy stored in the capacitor,

U1 = (½) C1 (V1)2 = (½)×600× 10-12× (200)2

= 12×10-6 J

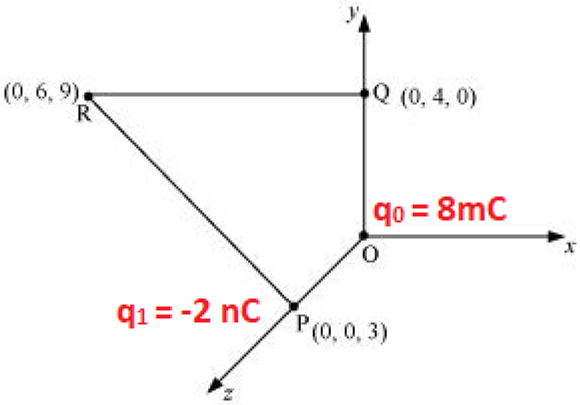
$$\therefore \text{ Energy loss } = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1+C_2)}$$

$$= \frac{36\times10^{-20}\times4\times10^{4}}{2\times12\times10^{-10}}$$

$$= 6 \times 10^{-6} \text{ J}$$

2.12 A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of –2 × 10–9 C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Ans - The electrostatic force's work on a charge is independent of the path the charge takes. It just depends on the charge's starting and ultimate placements.



Since, Work is independent on the path followed we concentrate on point P and Q only.

Potential at any point is given by, $V = \frac{q}{4\pi\epsilon_0 d} \text{N m C}^{-1}$

Where,

q = charge

d = distance from origin

ϵ_0 = permittivity of space

and, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

2.13 A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Ans -

$$V' = 8V = 8 \times \frac{q}{4\pi \epsilon_0 r}$$

$$= \frac{8q}{4\pi \epsilon_0 \frac{\sqrt{3}}{2} b} = \frac{4q}{\sqrt{3}\pi \epsilon_0 b}$$

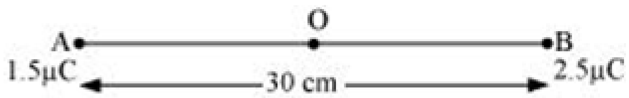
2.14 Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:
(a) at the mid-point of the line joining the two charges, and
(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.
Ans -

Potential at any point is given by, $V = \frac{q}{4\pi\epsilon_0 d}$ N m C^{-1}

Electric field generated by a charge, $E = \frac{q}{4\pi\epsilon_0 d^2}$ N C^{-1}

Where,
q = charge
d = distance from origin
ϵ_0 = permittivity of space
and, $\frac{1}{4\pi\epsilon_0} = 9\times10^9$ N m^2 C^{-2}

(a)Figure below represents the given situation,



Let the potential at O (midpoint) be V and Electric field be E.
We have,

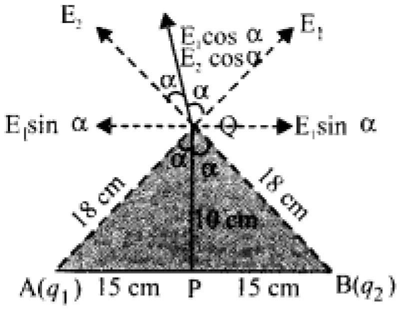
$$\therefore V = \frac{1.5\times10^{-6}\times9\times10^{9}}{0.15} + \frac{2.5\times10^{-6}\times9\times10^{9}}{0.15} = 2.4\times10^{5} \text{ N m C}^{-1}$$

and,
E = electric field due to charge 2.5μC – electric field due to charge 1.5μC

$$\therefore E = \frac{2.5\times10^{-6}\times9\times10^{9}}{0.15^2} - \frac{1.5\times10^{-6}\times9\times10^{9}}{0.15^2}$$

$\Rightarrow E = 4\times10^{5}$ N C^{-1}

(b)



(*i*) Now, potential at Q due to the system of charges

$$V_Q = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{AQ} + \frac{q_2}{BQ}\right]$$

$$= 9 \times 10^9 \left[\frac{1{\cdot}5 \times 10^{-6}}{0{\cdot}18} + \frac{2{\cdot}5 \times 10^{-6}}{0{\cdot}18}\right] = 2 \times 10^5 \text{V}$$

(*ii*) $$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{(AQ)^2} = \frac{9 \times 10^9 \times 1{\cdot}5 \times 10^{-6}}{(0{\cdot}18)^2}$$

$$= 0{\cdot}42 \times 10^6 \, \text{Vm}^{-1}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(BQ)^2} = \frac{9 \times 10^9 \times 2{\cdot}5 \times 10^{-6}}{(0{\cdot}18)^2}$$

$$= 0{\cdot}69 \times 10^6 \, \text{Vm}^{-1}$$

Net electric field along Y-axis, $E_y = E_1 \cos\alpha + E_2 \cos\alpha$ (along $+y$-axis)

Net electric field along X-axis, $E_x = (E_2 \sin\alpha - E_1 \sin\alpha)$ (along $-x$-axis)

2.15 A spherical conducting shell of inner radius r 1 and outer radius r 2 has a charge Q.
(a) A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?
(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Ans - (a) Charge placed at the centre of a shell is $+q$. Hence, a charge of magnitude $-q$ will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is $-q$.

Surface charge density

$$\sigma_1 = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{-q}{4\pi r_1^2} \qquad \ldots (i)$$

A charge of +q is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is $Q + q$. Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \quad \text{... } (ii)$$

(b) Yes

Even if the shell is not spherical and has any irregular shape, the electric field intensity inside a cavity is zero. Consider a closed loop with some of it inside the conductor and some of it inside the cavity along a field line. Because there is no field inside the conductor, there is no net work done by the field when carrying a test charge over a closed loop. Therefore, regardless of shape, the electric field is zero.

2.16 (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by 2 1 0 ( ) ˆ σ ε E E− • n = where nˆ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of nˆ is from side 1 to side 2.) Hence, show that just outside a conductor, the electric field is σ nˆ /ε0 .
(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For
(a), use Gauss's law. For,
(b) use the fact that work done by electrostatic field on a closed loop is zero.]
Ans - (a) The charged body has an E1 electric field on one side and an E2 electric field on the opposite side. The electric field owing to one surface of an infinite plane charged body is given by, if the body has uniform thickness.

$$\vec{E}_1 = -\frac{\sigma}{2\varepsilon_0}\hat{n}$$

Electric field intensity on the right side of the sheet,

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

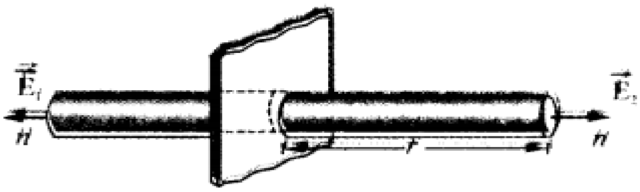
∴ Discontinuity in the normal component of the field from one side to other side is

$$\vec{E}_2 - \vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{n} + \frac{\sigma}{2\varepsilon_0}\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

or $$\left(\vec{E}_2 - \vec{E}_1\right)\cdot\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}\cdot\hat{n}$$

Since inside the conductor, $\vec{E}_1 = 0$, therefore,

$$\vec{E}_1 = \vec{E}_2 = \frac{\sigma}{\varepsilon_0}\hat{n}.$$



2.17 A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Ans - A cylinder P has linear charge density, λ, length l, and radius r1

The charge on cylinder P, q = XL A hollow co-axial conducting cylinder of length / and radius r2 surrounds the cylinder P. Charge on cylinder Q = – q.

Let q be the total charge on the cylinder,

$$\therefore \Phi = E(2\pi d)L = \frac{\sigma}{\epsilon_0}$$

Where, q = charge of the inner sphere of the outer cylinder

ϵ_0 = permittivity of space

Thus,

$$E(2\pi d)L = \frac{\lambda L}{\epsilon_0}$$

$$\therefore E = \frac{\lambda}{2\pi d\epsilon_0}\ NC^{-1}$$

Hence, the electric field between cylinders, $E = \frac{\lambda}{2\pi d\epsilon_0}\ NC^{-1}$

2.18 In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å: (a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?
(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?
Ans -
Given,

The distance between electron-proton of hydrogen atom = 0.53 Å = 0.53× 10-10m

$$(a)\ \text{P.E., U} = \frac{1}{4\pi \epsilon_0}\frac{q_1 q_2}{r}$$

$$= \frac{9\times10^9\times(-1{\cdot}6\times10^{-19})(1{\cdot}6\times10^{-19})}{0{\cdot}53\times10^{-10}}$$

$$= -43{\cdot}47 \times 10^{-19}\ \text{J}$$

(b) Given, Kinetic energy is half of the potential energy in (a).
∴ Kinetic energy, K = 0.5× (27.2) = 13.6 eV
Thus, Total energy of electron = -27.2 + 13.6 = -13.6 eV
Amount of work require to free the electron, A = Increase in energy the electron.
∴ A = 0-(-13.6) = 13.6 eV
(c) If we take potential energy Zero at 1.06 Å separation, then the potential energy of the system,

$$P = -9\times10^9\ \text{N m}^2\ \text{C}^{-2}\left[\frac{(1.6\times10^{-19}\text{C})\times(1.6\times10^{-19}\text{C})}{1.06\times10^{-10}\ \text{m}}\right]$$

$\Rightarrow P = -21.74\times10^{-19}$ J
In electron volts, the potential energy is given as,
P = -13.585 eV.
(1eV = 1.6×10^{-19} J)

∴ The amount of work done to free the electron in this case,
W = -27.17- (-13.585) = -13.585 eV

2.19 If one of the two electrons of a H2 molecule is removed, we get a hydrogen molecular ion H+ 2 . In the ground state of an H+ 2 , the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Ans - Charge on proton 1, $q1 = 1.6 \times10{-}19$ C

Charge on proton 2, $q2 = 1.6 \times 10-19$ C

Charge on electron, $q3 = -1.6 \times 10-19$ C

Distance between protons 1 and 2, $d1 = 1.5 \times 10-10$ m

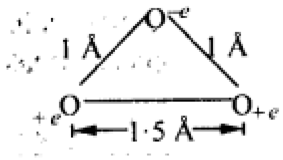
Distance between proton 1 and electron, $d2 = 1 \times 10-10$ m

Distance between proton 2 and electron, $d3 = 1 \times 10-10$ m

The potential energy at infinity is zero

Using P.E. $= \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2}$ for each pair of charges

and then adding up, we get the total P.E.



$$\frac{1}{4\pi\varepsilon_0}\left[\frac{e \times e}{1\cdot 5 \times 10^{-10}} - \frac{e \times e}{10^{-10}} - \frac{e \times e}{10^{-10}}\right]$$

$$= \frac{(9 \times 10^9)(1\cdot 6 \times 10^{-19})^2}{10^{-10}}\left[\frac{1}{1\cdot 5} - 1 - 1\right]$$

$= -19\cdot 2$ eV

Zero of P.E. is taken as infinity.

2.20 Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Ans - Since a flat surface can be compared to a spherical surface with a big radius and a pointed portion to a spherical surface with a small radius, it follows that the pointed surface will have a higher electric charge density.

Using $E = \frac{dV}{dr}$, we get $V = Er$

Then $V = E_1 a$ and $V = E_2 b$

i.e. $E_1 a = E_2 b$ or $\frac{E_1}{E_2} = \frac{b}{a}$.

2.21 Two charges –q and +q are located at points (0, 0, –a) and (0, 0, a), respectively.

(a) What is the electrostatic potential at the points (0, 0, z) and (x, y, 0) ?

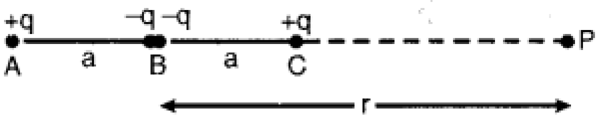
(b) Obtain the dependence of potential on the distance r of a point from the origin when r/a >> 1.
(c) How much work is done in moving a small test charge from the point (5,0,0) to (–7,0,0) along the x-axis? Does the answer change if the path of the test charge between the same points is not along the x-axis?

Ans - (a) Charge $-q$ is located at $(0, 0, -a)$ and charge $+q$ is located at $(0, 0, a)$.
Hence, they form a dipole. Point $(0, 0, z)$ is on the axis of this dipole and point $(x, y, 0)$ is normal to the axis of the dipole.
Hence, electrostatic potential at point $(x, y, 0)$ is zero.
Electrostatic potential at point $(0, 0, z)$ is given by

$$V = \frac{1}{4\pi \in_0}\left(\frac{q}{z-a}\right) + \frac{1}{4\pi \in_0}\left(-\frac{q}{z+a}\right)$$
$$= \frac{q(z+a-z+a)}{4\pi \in_0 (z^2-a^2)}$$
$$= \frac{2qa}{4\pi \in_0 (z^2-a^2)} = \frac{p}{4\pi \in_0 (z^2-a^2)}$$

(b) More than half of the distance between the two charges is covered by the distance r. The potential (V) is therefore inversely proportional to the square of the distance,
(c) zero The answer does not change because, in electrostatics, the work done does not depend upon the actual path, it simply depends upon the initial and final positions.

2.22 Figure 2.32 shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



Ans -

$$V = \frac{1}{4\pi \in_0}\left[\frac{q}{XP} - \frac{2q}{YP} + \frac{q}{ZP}\right]$$
$$= \frac{1}{4\pi \in_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$$
$$= \frac{q}{4\pi \in_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$
$$= \frac{q}{4\pi \in_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2-a^2)}\right] = \frac{q}{4\pi \in_0}\left[\frac{2a^2}{r(r^2-a^2)}\right]$$
$$= \frac{2qa^2}{4\pi \in_0 r^3\left(1-\frac{a^2}{r^2}\right)}$$

(2) The potential is of the 1/r2 type because of the electric dipole.
(3) The potential is of the 1/r type as a result of an electric monopole.

2.23 An electrical technician requires a capacitance of 2 µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Ans - Potential difference across the circuit = 1kV = 1000V

Capacitance of each capacitor = 1 µF

Potential difference each capacitor can withstand = 400V

Capacitance required across the circuit = 2 µF

$$400 \times n = 10^3$$

i.e. $$n = \frac{10^3}{400} = 2{\cdot}5 \quad i.e. \quad 3$$

In parallel

$$\frac{1}{C} = \frac{1}{1}+\frac{1}{1}+\frac{1}{1} = 3$$

i.e. $$C = \frac{1}{3}\mu F$$

Total capacitance of m rows, $C_{eq} = mC$

i.e. $$m = \frac{C_{eq}}{C} = \frac{2}{1/3} = 6$$

$$\text{Total Capacitors} = m \times n = 6 \times 3 = 18$$

2.24 What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realise from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Ans - Capacitance of a parallel capacitor, *V* = 2 F

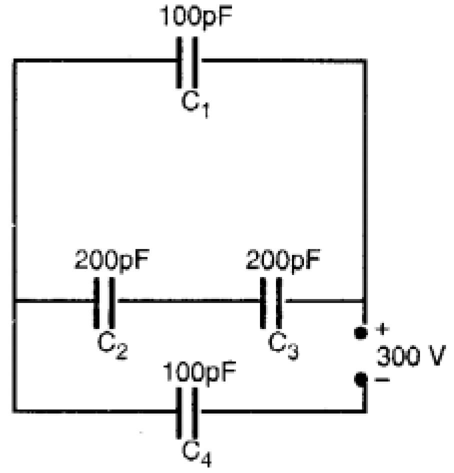
Distance between the two plates, *d* = 0.5 cm = 0.5 × 10−2 m

Using $C = \frac{\epsilon_0 A}{d}$, we get

$$A = \frac{Cd}{\epsilon_0} = \frac{2 \times 0{\cdot}5 \times 10^{-2}}{8{\cdot}854 \times 10^{-12}}$$

$$= 1{\cdot}13 \times 10^9 \text{ m}^2.$$

2.25 Obtain the equivalent capacitance of the network in Fig. 2.33. For a 300 V supply, determine the charge and voltage across each capacitor.



Ans - Capacitance of capacitor *C*1 is 100 pF.

Capacitance of capacitor *C*2 is 200 pF.

Capacitance of capacitor *C*3 is 200 pF.

Capacitance of capacitor *C*4 is 100 pF.

Supply potential, *V* = 300 V

Potential difference across C4 is in the ratio 2:1
i.e., 200 V
.’. Charge on C4 = C4V4
= 100 x 200 x 10-12 = 2 x 10-8 C
Potential difference across C1= 100 V
Charge on C1 = C1x V1
=100 x 100 x 10-12 = 1 x 10-8 C
Potential difference across C2 and C3 is 50 v each
∴ Charge on C2 or C3 = C2V2
= 200 x 50 x 10-12 = 10-8 C.

2.26 The plates of a parallel plate capacitor have an area of 90 cm2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.
(a) How much electrostatic energy is stored by the capacitor?
(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Ans - Area of the plates of a parallel plate capacitor, $A = 90\ cm2 = 90 \times 10{-}4\ m2$

Distance between the plates, $d = 2.5\ mm = 2.5 \times 10{-}3\ m$

Using $C = \frac{\epsilon_0 A}{d}$,

we get $C = \frac{8{\cdot}854 \times 10^{-12} \times 90 \times 10^{-4}}{2{\cdot}5 \times 10^{-3}}$

$= 3{\cdot}187 \times 10^{-11}$ F

Work done,

$$W = \frac{1}{2} CV^2$$

$$= \frac{1}{2} \times 3{\cdot}187 \times 10^{-11} \times 400^2$$

$= 2{\cdot}55 \times 10^{-6}$ J.

Energy per unit volume,

$U = 0{\cdot}113\ J\ m^{-3}$

Energy per unit volume,

$$U = \frac{1}{2}\frac{CV^2}{Ad}$$

But $E = \frac{V}{d}$ *i.e.* $V = Ed$

∴ Energy per unit volume,

$$U = \frac{1}{2}\frac{CE^2d^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 A}{d}\frac{E^2d^2}{Ad}$$

Relation between U and E is,

$$U = \frac{1}{2}\epsilon_0 E^2.$$

2.27 A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Ans - given,

Capacitance= 4uf
V=200
Uncharged capacitor= 2uf

Capacitance of a charged capacitor(C) = 4 μF = 4×10^{-6}F
Voltage supplied to the capacitor (V) = 200 V
Electrostatic energy of the capacitor (E) = $\frac{1}{2}CV^2 = \frac{1}{2}\times 4\times 10^{-6}\times (200)^2 = 8\times10^{-2}$ J
Capacitance of an uncharged capacitor (C′) = 2 μF = 2×10^{-6}F
When both the capacitors are connected in a circuit, then initial charge on charged capacitor is equal to the final charge on both the capacitors in circuit. (According to law of conservation of charges)
Since, Charge = Voltage ×Capacitence
Therefore, C×V = (C + C′) ×V′, where V′ is the voltage in the circuit when both capacitors are connected.
$4\times10^{-6}F\times200V = (4\times10^{-6}F + 2\times10^{-6}F)\times V'$

$$V' = \frac{4\times10^{-6}\times200V}{6\times10^{-6}F}$$

$$\Rightarrow V = \frac{400}{3}V$$

Now, Electrostatic energy for the combination of two capacitors = 1/2 (C + C′)V′²

$$= \frac{1}{2}\times 6\times 10^{-6}\times\left(\frac{400}{3}\right)^2 J$$

2.28 Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.

Ans - work done by the force to do so = $F\Delta x$

As a result, the potential energy of the capacitor increases by an amount given as $uA\Delta x$.

Where,

u = Energy density

A = Area of each plate

d = Distance between the plates

V = Potential difference across the plates

$$F\Delta x = uA\Delta x$$

$$F = uA = \left(\frac{1}{2}\epsilon_0 E^2\right)A$$

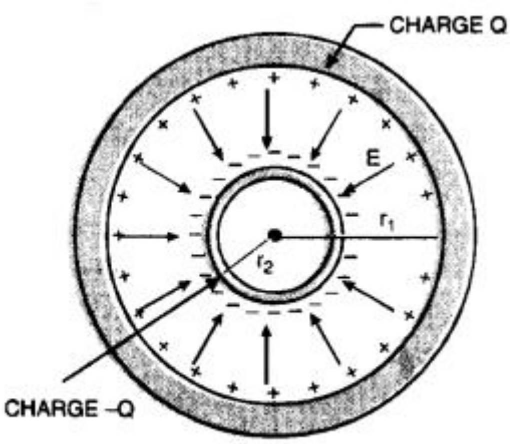
$$\text{or } F = \frac{1}{2}\varepsilon_0 E^2 A = \frac{1}{2}(\varepsilon_0 A\ E)E = \frac{1}{2}\left(\varepsilon_0 A\frac{V}{d}\right)E$$

$$\text{since } \frac{\varepsilon_0 A}{d} = C$$

$$\because F = \frac{1}{2}(C\ V)E \quad \text{or } F = \frac{1}{2}QE \qquad (\because Q = CV)$$

Since electric field inside a conductor is zero and outside the conductor, the electric field is E. Therefore, average of electric field $\left(\frac{0+E}{2} = \frac{E}{2}\right)$ contributes to the force.

2.29 A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.34). Show that the capacitance of a spherical capacitor is given by



$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r 1 and r 2 are the radii of outer and inner spheres, respectively.
Ans -

Radius of the outer shell = *r*1

Radius of the inner shell = *r*2

The inner surface of the outer shell has charge +Q.

The outer surface of the inner shell has induced charge $-Q$

Radius of the outer shell = r_1
Radius of the inner shell = r_2
Charge on the inner surface of the outer shell = Q
Charge on the outer surface of the inner shell = -Q
Potential difference between the two shells, $\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

$$\Delta V = \frac{Q(r_1 - r_2)}{4\pi\varepsilon_0 r_2 r_1}$$

Where, ε_0 = Absolute Permittivity of free space = $8.85\times10^{-12} C^2N^{-1}m^{-2}$

Since, Capacitance, $C = \frac{\text{Charge}(Q)}{\text{Potential difference}(V)}$

$$\Rightarrow C = \frac{Q}{\frac{Q(r_1 - r_2)}{4\pi\varepsilon_0 r_2 r_1}}$$

$$\therefore C = \frac{4\pi\varepsilon_0 r_2 r_1}{r_1 - r_2}$$

Hence, proved.

2.30 A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.
(a) Determine the capacitance of the capacitor.
(b) What is the potential of the inner sphere?
(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.
Ans -

(a) Using $C = 4\pi \epsilon_0 k \frac{ab}{b-a}$, we get

$$C = \frac{1}{9\times10^9} \frac{32\times12\times10^{-2}\times13\times10^{-2}}{(13\times10^{-2})-(12\times10^{-2})}$$

$$= 5.547 \times 10^{-9} \text{ F}$$

(b) $$V = \frac{q}{C} = \frac{2.5\times10^{-6}}{5.547\times10^{-9}} = 450.7 \text{ V}$$

(c) $$C' = 4\pi\epsilon_0 r$$

$$= \frac{1}{9\times10^9} \times 12 \times 10^{-12}$$

$$= 1.33 \times 10^{-11} \text{ F}$$

$$\frac{C}{C'} = \frac{5.547\times10^{-9}}{1.333\times10^{-11}} = 416.$$

Clearly C′ is small because there is no nearby earthed conducting plate.

2.31 Answer carefully:

(a) Two large conducting spheres carrying charges Q1 and Q2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by Q1 Q2 /4πε0 r 2 , where r is the distance between their centres?

(b) If Coulomb's law involved 1/r 3 dependence (instead of 1/r 2), would Gauss's law be still true ?

(c) A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

(d) What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

(e) We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

(f) What meaning would you give to the capacitance of a single conductor?

(g) Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Ans - (a) No. Coulomb's law, which is the indicated relationship, holds true for point charges. The charge distribution on the spheres in the current scenario becomes non-uniform when they are moved closer to one another.

(b) Gauss's law will not be true, if Coulomb's law involved $1/r^3$ dependence, instead of$1/r^2$, on r.

(c) Definitely not. Charged particles do not always move in a straight line with the field. It does this in a level playing field. The field indicates the acceleration direction, not the direction of velocity in general.

d) The work that a nucleus' field does whenever an electron completes an orbit, whether it be circular or elliptical, is zero.

(e) No

Electric field is discontinuous across the surface of a charged conductor. However, electric potential is continuous.

(f) The single conductor and the other conductor can form a condenser at infinity. As a result, the definition of charge storage is retained.
(g) Water has an unsymmetrical space as compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica.

2.32 A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).
Ans - Radius of outer cylinder(R) = 1.5cm = 0.015m

Radius of inner cylinder(r) = 1.4cm = 0.014m

$$C = \frac{2\pi \epsilon_0 l}{2{\cdot}303 \times \log_{10} \frac{b}{a}}$$

$$\Rightarrow C = \frac{2\pi(8{\cdot}854 \times 10^{-12})(15 \times 10^{-2})}{2{\cdot}303 \log_{10}\left(\frac{1{\cdot}5 \times 10^{-2}}{1{\cdot}4 \times 10^{-2}}\right)}$$

$$= 1{\cdot}21 \times 10^{-10} \text{ F.}$$

Potential of inner cylinder

$$V = \frac{q}{C} = \frac{3{\cdot}5 \times 10^{-6}}{1{\cdot}21 \times 10^{-10}} = 2{\cdot}89 \times 10^4 \text{ V.}$$

2.33 A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 107 Vm–1. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?
Ans - Dielectric constant of a material(εr) = 3

Dielectric strength = 107V/m

Using $E = \frac{dV}{dr}$

i.e. $E = \frac{V}{r}$, we get

$$r = \frac{V}{E} = \frac{1000}{0{\cdot}1 \times 10^7} = 10^{-3} \text{ m}$$

Using $C = \frac{\epsilon_0 \epsilon_r A}{d}$, we get

$$A = \frac{Cd}{\epsilon_0 \epsilon_r} = \frac{Cr}{\epsilon_0 \epsilon_r}$$

$$= \frac{(50 \times 10^{-12})(10^{-3})}{8{\cdot}854 \times 10^{-12} \times 3}$$

$$= 19 \text{ cm}^2.$$

2.34 Describe schematically the equipotential surfaces corresponding to
(a) a constant electric field in the z-direction,
(b) a field that uniformly increases in magnitude but remains in a constant (say, z) direction,
(c) a single positive charge at the origin, and
(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane

Ans - (a) The equipotential surfaces are equidistant planes that are parallel to the x-y plane.

(b) Equipotential surfaces are planes that are parallel to the x-y plane, with the caveat that as the planes come closer, the field gets stronger.

(c) Equipotential surfaces are concentric spheres with their centres at the origin.

(d) The equipotential surface is a geometry that periodically changes close to the specified grid. At a further distance, this shape progressively transforms into the shape of planes parallel to the grid.

2.35 A small sphere of radius r 1 and charge q1 is enclosed by a spherical shell of radius r 2 and charge q2 . Show that if q1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q2 on the shell is.

Ans - A conductor's outside surface is where charge is located. As a result, the charge on the inner sphere will move through the conducting wire and into the shell. Additionally, according to Gauss' law, an electric field does not exist inside of a Gaussian surface, and charges contained within a closed surface only contribute to the field. Q2 is hence irrelevant in this situation. It's good, and the possibility of a difference is good too.

2.36 Answer the following:
(a) The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm–1. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)
(b) A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1m2 . Will he get an electric shock if he touches the metal sheet next morning?
(c) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?
(d) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning? (Hint: The earth has an electric field of about 100 Vm–1 at its surface in the downward direction, corresponding to a surface charge density = –10–9 C m–2. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)
Ans - (A) The surface created by our body and the ground is equipotential. The basic equipotential surfaces of open air alter as we move outside, keeping our head and the ground at the same potential.

(b) Yes. Depending on the capacitance of the capacitor, the steady discharge current in the atmosphere progressively charges up the aluminium sheet and raises its voltage to some level (formed by the sheet, slab, and the ground).

(c) Lightning and thunderstorms discharge charged atmospheric particles through areas of fair weather all around the world. On the whole, the two opposing currents are in equilibrium.

(d) The lightning's use of light energy; the thunder's use of heat and sound energy.