EXERCISE QUESTIONS

CHAPTER - 13 NUCLEI

13.1 (a) Two stable isotopes of lithium 6 3 Li and 7 3 Li have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.
(b) Boron has two stable isotopes, 10 5B and 11 5B. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of 10 5B and 11 5 B.

Ans - Mass of lithium isotope ${}^{6}_{3}Li$, $m_1 = 6.01512$ u

Mass of lithium isotope ${}^{7}_{3}Li$, $m_2 = 7.01600$ u

$$= \frac{6{\cdot}01512 \times 7{\cdot}5 + 7{\cdot}01600 \times 92{\cdot}5}{100} = \frac{45{\cdot}1134 + 648{\cdot}98}{100}$$

$= 6{\cdot}9409\, u$

(*b*) Let $x\%$ and $y\%$ be the abundances of ${}^{10}_{5}B$ and ${}^{11}_{5}B$ respectively.

$\therefore \quad x + y = 100 \quad \ldots(i)$

Also, Atomic weight

$$= \frac{x \times 10{\cdot}01294 + y \times 11{\cdot}00931}{100}$$

i.e. $10{\cdot}811 \times 100 = 10{\cdot}01294x + 11{\cdot}00931y$

i.e. $10{\cdot}01294x + 11{\cdot}00931y = 1081{\cdot}1 \quad \ldots(ii)$

Using equation (*i*), we get

$10{\cdot}01294(100 - y) + 11{\cdot}00931y = 1081{\cdot}1$

i.e. $1001{\cdot}294 + 0{\cdot}99637y = 1081{\cdot}1$

i.e. $y = 80{\cdot}09\%$

Then $x = 100 - 80{\cdot}09 = 19{\cdot}91\%$.

13.2 The three stable isotopes of neon: 20 21 22 10 10 Ne, Ne and Ne 10 have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Ans - Atomic mass of ${}^{21}_{10}Ne$, $m_2 = 20.99$ u

Abundance of ${}^{21}_{10}Ne$, $\eta_2 = 0.27\%$

Atomic mass of ${}^{22}_{10}Ne$, $m_3 = 21.99$ u

Abundance of ${}^{22}_{10}Ne$, $\eta_3 = 9.22\%$

Average atomic mass of neon

$$= \frac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99}{100}$$

$$= \frac{1809.29 + 5.67 + 202.75}{100} = 20.18 \text{ u.}$$

13.3 Obtain the binding energy (in MeV) of a nitrogen nucleus ( ) 14 7 N , given m ( ) 14 7 N =14.00307 u

Ans - Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

Mass defect, $\Delta m = Zm_H + (A - Z)\, m_n - m_N$

Here A = 14, Z = 7

$\therefore \quad \Delta m = 7 \times 1.00783 + (14 - 7) \times 1.00867 - 14.00307$

$= 7.05481 + 7.06069 - 14.00307$

$= 0.011243$ u

$\therefore$ B.E. $= \Delta m \times 931$ MeV

$= 0.11243 \times 931 = 104.7$ MeV

13.4 Obtain the binding energy of the nuclei 56 26Fe and 209 83 Bi in units of MeV from the following data:

m (56 26Fe) = 55.934939 u m (209 83 Bi) = 208.980388 u

Ans - Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

For ${}^{56}_{26}Fe$, mass defect

$$\Delta m = Z\, m_H + (A - Z)\, m_n - m\left({}^{56}_{26}Fe\right)$$

$$= 26 \times 1{\cdot}007825 + (56 - 26) \times 1{\cdot}008665 - 55{\cdot}934939 = 0{\cdot}528461 \text{ u.}$$

$$\therefore \quad B.E. = \Delta m \times 931{\cdot}5 \text{ MeV} = 492{\cdot}0 \text{ MeV.}$$

$\therefore$ Binding energy per nucleon

$$= \frac{B.E.}{A} = \frac{492{\cdot}0}{56} = 8{\cdot}79 \text{ MeV}$$

For ${}^{209}_{83}Bi$, mass defect is

$$\Delta m = Z m_H + (A - Z)\, m_n - m\left({}^{209}_{83}Bi\right)$$

$$= 83 \times 1{\cdot}007825 + (209 - 83) \times 1{\cdot}008665 - 208{\cdot}980388 = 1{\cdot}760877 \text{ u.}$$

$$\therefore \text{ Binding energy} = \Delta m \times 931{\cdot}5 \text{ MeV} = 1{\cdot}760877 \times 931{\cdot}5 = 1640{\cdot}26 \text{ MeV}$$

$\therefore$ Binding energy per nucleon

$$= \frac{B.E.}{A} = \frac{1640{\cdot}26}{209} = 7{\cdot}85 \text{ MeV}$$

Clearly, ${}^{56}_{26}Fe$ has more Binding energy per nucleon compared to ${}^{209}_{83}Bi$.

13.5 A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of 63 29Cu atoms (of mass 62.92960 u).

Ans - Mass of a proton, m_H = 1.007825 u

Mass of a neutron, m_n = 1.008665 u

∴ Δm' = 29 × 1.007825 + 34 × 1.008665 − 62.9296

= 0.591935 u

$$\therefore \Delta m = 29 \times 1{\cdot}00783 + 34 \times 1{\cdot}00867 - 62{\cdot}92960 = 0{\cdot}59225 \text{ u.}$$

$$\therefore \quad \text{Binding energy} = \Delta m \times 931 \text{ MeV} = 0{\cdot}59225 \times 931 = 551{\cdot}38 \text{ MeV.}$$

Now 1 gram mole of copper contains $6{\cdot}02 \times 10^{23}$ atoms.

∴ mass of 63 g has $6{\cdot}02 \times 10^{23}$ atoms.

∴ mass of 3 g of copper has $\frac{6{\cdot}02 \times 10^{23} \times 3}{63}$ atoms.

Now Binding energy of one atom is 551·38 MeV

∴ Binding energy of $\frac{6{\cdot}02 \times 10^{23} \times 3}{63}$ atoms

$$= \frac{551{\cdot}38 \times 6{\cdot}02 \times 10^{23} \times 3}{63}$$

$= 1{\cdot}58 \times 10^{25}$ MeV.

13.6 Write nuclear reaction equations for
(i) α-decay of 226 88 Ra
(ii) α-decay of 242 94 Pu
(iii) β– -decay of 32 15 P
(iv) β– -decay of 210 83 Bi
(v) β+ -decay of 11 6 C
(vi) β+ -decay of 97 43 Tc
(vii) Electron capture of 120 54 Xe

Ans - This is a helium nucleus. - An electron (e for atom, and e+ for atom plus) is a nucleiand. A loss of two protons and four neutrons occurs in every -decay. A neutrino and one proton are lost in every +-decay, and the nucleus releases one. An antineutrino is emitted from the nucleus along with a gain of 1 proton in every -decay.

The various nuclear reactions for the situations above can be expressed as follows:

:

(i) $^{226}_{83}Ra \rightarrow ^{222}_{86}Rn + ^{4}_{2}He$

(ii) $^{242}_{94}Pu \rightarrow ^{238}_{92}U + ^{4}_{2}He$

(iii) $^{32}_{15}P \rightarrow ^{32}_{16}S + e^- + \bar{v}$

(iv) $^{210}_{83}Bi \rightarrow ^{210}_{84}Po + e^- + \bar{v}$

(v) $^{11}_{6}C \rightarrow ^{11}_{5}B + e^+ + v$

(vi) $^{97}_{43}Tc \rightarrow ^{97}_{42}Mo + e^+ + v$

(vii) $^{120}_{54}Xe + e^+ \rightarrow ^{120}_{53}I + v$

13.7 A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to
a) 3.125%,
b) 1% of its original value?

Ans - T years is the radioactive isotope's half-life.

The radioactive isotope's original concentration was N0.

(a) Following decay, the radioactive isotope's quantity equals N.

$$(a) \quad \frac{A}{A_0} = \frac{N}{N_0} = \frac{3{\cdot}125}{1000_2} = \frac{1}{32} = \left(\frac{1}{2}\right)^5$$

$$\text{Now} \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\therefore \quad n = 5 \text{ or } t = 5 \text{ T years}$$

$$(b) \quad \frac{A}{A_0} = \frac{N}{N_0} = \frac{1}{100}$$

$$\therefore \quad t = \frac{2{\cdot}303}{\lambda}\log\frac{N_0}{N} = \frac{2{\cdot}303\,T}{0{\cdot}093}\log 100$$

13.8 The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive 14 6 C present with the stable carbon isotope 12 6 C . When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of 14 6 C , and the measured activity, the age of the specimen can be approximately estimated. This is the principle of 14 6C dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Ans - Living carbon-containing stuff decays at a rate of 15 decays per minute (R).

Let N represent the total number of radioactive atoms in a typical carbon-containing substance.

Here $A_0 = 15$ decays/minute/g;

$A = 9$ decays/minute/g

$\therefore$ Using $A = A_0 e^{-\lambda t} \Rightarrow 9 = 15 e^{-\lambda t}$

or $e^{\lambda t} = \frac{15}{9} = \frac{5}{3}$

Taking natural log of both sides

$$\log_e e^{\lambda t} = \log_e \left(\frac{5}{3}\right)$$

$$\text{or} \quad \lambda t = 2 \cdot 303 \left(\log \frac{5}{3}\right)$$

$$= 2 \cdot 303 \,[\log 5 - \log 3]$$

$$\therefore \quad \lambda t = 2 \cdot 303 \,[0 \cdot 6990 - 0 \cdot 4771]$$

$$= 2 \cdot 303 \times 0 \cdot 2219 = 0 \cdot 511$$

$$\therefore \quad t = \frac{0 \cdot 511}{\lambda} = \frac{0 \cdot 511}{0 \cdot 693} \times T_{1/2}$$

$$\left[\because \lambda = \frac{0 \cdot 693}{T_{1/2}}\right]$$

$$= \frac{0 \cdot 511}{0 \cdot 693} \times 5730 = 4225 \text{ years.}$$

13.9 Obtain the amount of 60 27Co necessary to provide a radioactive source of 8.0 mCi strength. The half-life of 60 27Co is 5.3 years.

Ans - The radioactive source's potency is shown as follows:

8.0mCi = dN/dt

Where,

N is the quantity of atoms needed.

$$A = \lambda N = \frac{0{\cdot}693}{T} N$$

$$N = \frac{AT}{0{\cdot}693}$$

$$= \frac{2{\cdot}96 \times 10^8 \times 1{\cdot}67 \times 10^8}{0{\cdot}693}$$

$$= 7{\cdot}13 \times 10^{16}$$

Now $6{\cdot}023 \times 10^{23}$ atoms of cobalt have mass = 60 g

$\therefore$ $7{\cdot}13 \times 10^{16}$ atoms of cobalt have mass

$$= \frac{60 \times 7{\cdot}13 \times 10^{16}}{6{\cdot}023 \times 10^{23}} = 7{\cdot}1 \times 10^{-6} \text{ g}$$

13.10 The half-life of 90 38Sr is 28 years. What is the disintegration rate of 15 mg of this isotope?

Ans - Half life of $^{90}_{38}Sr$, $T_{\frac{1}{2}}$ = 28 years

= 28 × 365 × 24 × 60 × 60

= 8.83×10^8 s

Number of atoms in 15 mg $^{90}_{38}Sr$

$$= \frac{6 \cdot 02 \times 10^{23} \times 15 \times 10^{-3}}{90} = N$$

Disintegration constant, λ

$$= \frac{0 \cdot 693}{T} = \frac{0 \cdot 693}{28 \times 365 \times 24 \times 3600} s^{-1}$$

$\therefore$ Rate of disintegration $\left(\frac{dN}{dt}\right) = \lambda N$

$$= \frac{0 \cdot 693 \times 6 \cdot 02 \times 10^{23} \times 15 \times 10^{-3}}{28 \times 365 \times 24 \times 3600 \times 90}$$

$= 7 \cdot 87 \times 10^{10}$ Bq.

13.11 Obtain approximately the ratio of the nuclear radii of the gold isotope 197 79 Au and the silver isotope 107 47 Ag .

Ans - Gold mass number: AAu = 197

Silver mass number is 107 (AAg).

The following relationship exists between the two nuclei's mass numbers and their ratio of radii:

$$R \propto A^{1/3}$$

$$\therefore \frac{R_{Au}}{R_{Ag}} = \left(\frac{A_{Au}}{A_{Ag}}\right)^{\frac{1}{3}} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = (1 \cdot 84)^{\frac{1}{3}}$$

$$= 1 \cdot 23.$$

13.12 Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of
(a) 226 88 Ra and
(b) 220 86 Rn .
Given m (226 88 Ra) = 226.02540 u, m (222 86 Rn) = 222.01750 u, m (220 86 Rn) = 220.01137 u, m (216 84 Po) = 216.00189 u.

Ans - Alpha particle decay of $^{226}_{88}\text{Ra}$ emits a helium nucleus. As a result, its mass number reduces to (226 − 4) 222 and its atomic number reduces to (88 − 2) 86. This is shown in the following nuclear reaction.

∴ Q-Value

$= M(^{226}_{88}\text{Ra}) - [M(^{222}_{86}\text{Rn}) + M(_2\text{He}^4)]$

$= 226{\cdot}02540 - 226{\cdot}0201$

$= 0{\cdot}0053\ \text{u} = 0{\cdot}0053 \times 931\ \text{MeV}$

$(\because 1\text{u} = 931\ \text{MeV})$

$= 4{\cdot}93\ \text{MeV}$

K.E. of α-particle

$= \left(\frac{A-4}{A}\right)Q = \left(\frac{222-4}{222}\right) \times 4{\cdot}93$

$= 4{\cdot}84\ \text{MeV}$

(*b*) $^{220}_{86}\text{Rn} \longrightarrow {}^{216}_{82}\text{Po} + {}_2\text{He}^4$

∴ Q-value

$= m(^{220}_{86}\text{Rn}) - [m(^{216}_{82}\text{Po}) + m(^4_2\text{He})]$

$= 220{\cdot}011373 - [216{\cdot}00189 + 4{\cdot}00260]$

$= 220{\cdot}011373 - 220{\cdot}00449 = 0{\cdot}006883\ \text{u}$

$= 0{\cdot}006883 \times 931\ \text{MeV} = 6{\cdot}41\ \text{MeV}$

K.E. of α-particle

$= \left(\frac{A-4}{4}\right)Q = \left(\frac{216-4}{216}\right) \times 6{\cdot}41$

$= 6{\cdot}29\ \text{MeV}.$

13.13 The radionuclide 11C decays according to 11 11 + 6 5 C B+ + : =20.3 min → e T v 1/2 The maximum energy of the emitted positron is 0.960 MeV.
Given the mass values:
m (11 6 C) = 11.011434 u and m (11 6B) = 11.009305 u, calculate Q and compare it with the maximum energy of the positron emitted.

Ans - Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}C$ nucleus is given as:

subtracted from atomic mass of $^{11}_{5}B$ (Because their atoms contain outside electrons). Therefore, in terms of the atomic mass m [*i.e.* $m\left(^{11}_{6}C\right)$ and $m\left(^{11}_{5}B\right)$]

$$Q = \left[m\left(^{11}_{6}C\right) - 6m_e - m\left(^{11}_{5}B\right) + 5m_e - m_e\right] \times 931{\cdot}5 \text{ MeV}$$

$$= \left[m\left(^{11}_{6}C\right) - m\left(^{11}_{5}B\right) - 2m_e\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [11{\cdot}011434 - 11{\cdot}009305 - 2 \times 0{\cdot}000548] \times 931{\cdot}5$$

$$= 0{\cdot}001033 \times 931{\cdot}5 = 0{\cdot}962 \text{ MeV}.$$

This energy is comparable to actual energy released in the decay process.

13.14 The nucleus 23 10 Ne decays by β– emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted.
Given that:
m (23 10 Ne) = 22.994466 u m (23 11 Na) = 22.989770 u.

Ans -

The emitted nucleus is way heavier than the $/beta$ particle and the energy of the antineutrino is also negligible and therefore the maximum energy of the emitted electron is equal to the Q value.The emitted nucleus is way heavier than the $/beta$ particle and the energy of the antineutrino is also negligible and therefore the maximum energy of the emitted electron is equal to the Q value.

The decay process of $^{23}_{10}Ne$ is

$$^{23}_{10}Ne \longrightarrow {}^{23}_{11}Na + {}^{0}_{-1}e + \bar{\nu} + Q$$

If whole of energy is carried by β particle, then

$$Q = \left[m_N\left(^{23}_{10}Ne\right) - m_N\left(^{23}_{11}Na\right) - m_e\right] \times 931 \cdot 5 \text{ MeV}$$

where $m_N\left(^{23}_{10}Ne\right)$ and $m_N\left(^{23}_{11}Na\right)$ are nuclear masses of $^{23}_{10}Ne$ and $^{23}_{11}Na$ respectively. If $m\left(^{23}_{10}Ne\right)$ and $m\left(^{23}_{11}Na\right)$ are atomic masses of $^{23}_{10}Ne$ and $^{23}_{11}Na$ respectively, then

$$m_N\left(^{23}_{10}Ne\right) = m\left(^{23}_{10}Ne\right) - 10m_e$$

(∵ atom of Ne contains 10 electrons)

and $m_N\left(^{23}_{11}Na\right) = m\left(^{23}_{11}Na\right) - 11m_e$

(∵ atom of Na contains 11 electrons)

$$\therefore Q = \left[m\left(^{23}_{10}Ne\right) - 10m_e - m\left(^{23}_{11}Na\right) + 11m_e - m_e\right] \times 931 \cdot 5$$

$$= \left[m\left(^{23}_{10}Ne\right) - m\left(^{23}_{11}Na\right)\right] \times 931 \cdot 5 \text{ MeV}$$

$$= [22 \cdot 994466 - 22 \cdot 989770] \times 931 \cdot 5 \text{ MeV}$$

$$= 0 \cdot 004689 \times 931 \cdot 5 = 4 \cdot 37 \text{ MeV}.$$

13.15 The Q value of a nuclear reaction A + b → C + d is defined by Q = [mA + mb – mC – md]c2 where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) 13 2 2 11 1 1 H+ H H+ H →

(ii) 12 12 20 4 6 6 10 2 C+ C Ne+ He → Atomic masses are given to be m (2 1 H) = 2.014102 u m (3 1 H) = 3.016049 u m (12 6 C) = 12.000000 u m (20 10 Ne) = 19.992439 u

Ans - *The given nuclear reaction is:*

$$^{1}_{1}H + {}^{3}_{1}H \longrightarrow {}^{2}_{1}H + {}^{2}_{1}H$$

It is given that:

(*i*) The given reaction is

$$ {}_1^1H + {}_1^3H \longrightarrow {}_1^2H + {}_1^2H $$

$$ \therefore Q = [m_N({}_1^1H) + m_N({}_1^3H) - m_N({}_1^2H) - m_N({}_1^2H)] \times c^2 \quad ...(1) $$

where m_N refers to nuclear masses.

$\therefore$ if m refers to atomic masses, then

$$ m({}_1^1H) = m_N({}_1^1H) + m_e $$

$$ \Rightarrow \quad m_N({}_1^1H) = m({}_1^1H) - m_e $$

Similarly, $m_N({}_1^3H) = m({}_1^3H) - m_e$,

$m_N({}_1^2H) = m({}_1^2H) - m_e$

$$ \therefore m_N({}_1^1H) + m_N({}_1^3H) - m_N({}_1^2H) - m_N({}_1^2H) $$

$$ = m({}_1^1H) - m_e + m({}_1^3H) - m_e $$

$$ - m({}_1^2H) + m_e - m({}_1^2H) + m_e $$

$= m({}_1^1H) + m({}_1^3H) - m({}_1^2H) - m({}_1^2H)$

$= 1{\cdot}007825 + 3{\cdot}016049 - 2{\cdot}014102 - 2{\cdot}014102$

$= -0{\cdot}00433$ amu

$= -0{\cdot}00433 \times 1{\cdot}67 \times 10^{-27}$ kg

$= 7{\cdot}23 \times 10^{-30}$ kg

$\therefore$ Using equation (1),

$Q = -7{\cdot}23 \times 10^{-30} \times (3 \times 10^8)^2$

$= -6{\cdot}51 \times 10^{-13}$ J

$$ = -\frac{6{\cdot}51 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}} \text{ MeV} = -4{\cdot}06 \text{ MeV}. $$

Since Q value is negative, the reaction is endothermic.

(*ii*) The given reaction is

$$ {}_6^{12}C + {}_6^{12}C \longrightarrow {}_{10}^{20}Ne + {}_2^4He $$

$$ \therefore Q = [m_N({}_6^{12}C) + m_N({}_6^{12}C) - m_N({}_{10}^{20}Ne) - m_N({}_2^4He)]c^2 \quad ...(2) $$

where m_N refers to the nuclear mass. If m refers to the atomic mass, then

$$ m_N({}_6^{12}C) = m({}_6^{12}C) - 6m_e $$

$({}_6^{12}C$ has 6 electrons in its atom$)$

$$ m_N({}_{10}^{20}Ne) = m({}_{10}^{20}Ne) - 10m_e $$

and $\quad m_N({}_2^4He) = m({}_2^4He) - 2m_e$

$$ \therefore m_N({}_6^{12}C) + m_N({}_6^{12}C) - m_N({}_{10}^{20}Ne) - m_N({}_2^4He) $$

$$= m\left({}^{12}_{6}C\right) - 6m_e + m\left({}^{12}_{6}C\right) - 6m_e$$

$$- m\left({}^{20}_{10}Ne\right) + 10m_e - m\left({}^{4}_{2}He\right) + 2m_e$$

$$= m\left({}^{12}_{6}C\right) + m\left({}^{12}_{6}C\right) - m\left({}^{20}_{10}Ne\right) - m\left({}^{4}_{2}He\right)$$

$= 12{\cdot}000000 + 12{\cdot}000000 - 19{\cdot}992439 - 4{\cdot}002603 = 0{\cdot}004958$ u

$= 0{\cdot}004958 \times 1{\cdot}67 \times 10^{-27}$ kg

$= 8{\cdot}28 \times 10^{-30}$ kg

$\therefore$ from eqn. (2),

$Q = 8{\cdot}28 \times 10^{-30} \times (3 \times 10^{8})^{2}$

$= 7{\cdot}45 \times 10^{-13}$ J

$$= \frac{7{\cdot}45 \times 10^{-13}}{1{\cdot}6 \times 10^{-13}} \text{MeV} = 4{\cdot}66 \text{ MeV}$$

Since Q value is positive, the reaction is exothermic.

13.16 Suppose, we think of fission of a 56 26Fe nucleus into two equal fragments, 28 13 Al . Is the fission energetically possible? Argue by working out Q of the process. Given m (56 26Fe) = 55.93494 u and m (28 13 Al) = 27.98191 u.

Ans - The fission of ${}^{56}_{26}Fe$ can be given as:

$${}^{56}_{13}Fe \longrightarrow 2\ {}^{28}_{13}Al$$

It is given that:

Q-value of process $= m({}^{58}_{26}Fe - 2m({}^{28}_{13}Al)$

$= 55{\cdot}93494 - 2 \times 27{\cdot}98191$

$= -0{\cdot}02888$ u

$= -0{\cdot}02888 \times 931$

($\because$ 1 u = 931 MeV)

$= -26{\cdot}89$ MeV.

Since Q-value is negative, so the fission is not possible.

13.17 The fission properties of 239 94 Pu are very similar to those of 235 92 U . The average energy released per fission is 180 MeV. How much energy,in MeV, is released if all the atoms in 1 kg of pure 239 94 Pu undergo fission?

Ans - Amount of pure $^{239}_{94}Pu$, m = 1 kg = 1000 g

N_A= Avogadro number = 6.023×10^{23}

Mass number of $^{239}_{94}Pu$= 239 g

239 g of $^{239}_{94}Pu$ contains

$= 6 \cdot 023 \times 10^{23}$ nuclei

1000 g of $^{239}_{94}Pu$ contains

$$= \frac{6 \cdot 023 \times 10^{23}}{239} \times 1000$$

$= 2 \cdot 52 \times 10^{24}$ nuclei.

Energy released per fission of each nucleus

= 180 MeV

$\therefore$ Total energy released

$= 180 \times 2 \cdot 52 \times 10^{24}$ MeV

$= 4 \cdot 54 \times 10^{26}$ MeV.

13.18 A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much 235 92 U did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of 235 92 U and that this nuclide is consumed only by the fission process.

Ans - the fission of one nucleus of 23592U, energy generated is 200 MeV

Power P = 1000 MW

$= 1000 \times 10^6$ W $= 10^9$ W

Also, time t = 5·00 year

$= 5 \times 365 \times 24 \times 60 \times 60$ seconds

$= 1 \cdot 577 \times 10^8$ s

$\therefore$ Energy consumed = Pt

$= 10^9 \times 1 \cdot 577 \times 10^8 = 1 \cdot 577 \times 10^{17}$ J

We know energy generated per fission of $^{235}_{92}U$ = 200 MeV

$= 200 \times 1 \cdot 6 \times 10^{-13}$ J $= 3 \cdot 2 \times 10^{-11}$ J

$\therefore$ Number of total fissions occurred in five years

$$=\frac{1\cdot577\times10^{17}}{3\cdot2\times10^{-11}}=4\cdot93\times10^{27}$$

Now $6\cdot023\times10^{23}$ atoms (fissions) are produced by 235 g of ${}^{235}_{92}U$

$\therefore$ Mass of ${}^{235}_{92}U$ consumed in five years

$$=\frac{235}{6\cdot023\times10^{23}}\times4\cdot93\times10^{27}\ g$$

$= 1923543\ g \approx 1924\ kg$

$\therefore$ Initial mass of ${}^{235}_{92}U = 2\times1924$

$= 3848\ kg$

13.19 How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as 22 3 11 2 H+ H He+n+3.27 MeV →

Ans - Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

Power of lamp, P = 100 W and mass of deuterium = 2 kg

From the reaction given in the exercise, we find that 2 nuclei of ${}^{2}_{1}H$ combine to give 3·2 MeV of energy.

Now 2 g of ${}^{2}_{1}H$ is equivalent to $6\cdot023\times10^{23}$ nuclei

$\therefore$ 2 kg of ${}^{2}_{1}H$ is equivalent to $\frac{6\cdot023\times10^{23}}{2}\times2000$

$= 6\cdot023\times10^{26}$ nuclei

$\therefore$ The energy released in the fusion of $6\cdot023\times10^{26}$ nuclei,

$$E=\frac{3\cdot2}{2}\times6\cdot023\times10^{26}$$
$$=9\cdot64\times10^{26}\ MeV$$
$$=9\cdot64\times10^{26}\times1\cdot6\times10^{-13}\ J$$
$$=1\cdot54\times10^{14}\ J.$$

$\therefore$ Using $P=\frac{E}{t}$ or $t=\frac{E}{P}$, we get

$$t=\frac{1\cdot54\times10^{14}}{100}$$
$$=1\cdot54\times10^{12}\ s$$
$$=\frac{1\cdot54\times10^{12}}{365\times24\times60\times60}\ \text{years}$$
$$=4\cdot89\times10^{4}\ \text{years.}$$

13.20 Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Ans - Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$\therefore d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}$ m

(K.E. = kinetic energy)

When the two deutrons stop, their energy is totally potential energy U given by

$$U = \frac{1}{4\pi \epsilon_0} \frac{e.e}{2R} = \frac{1}{4\pi \epsilon_0} \cdot \frac{e^2}{2R}.$$

$\therefore$ Using law of conservation of energy,

$$2\,\text{K.E.} = \frac{1}{4\pi \epsilon_0} \frac{e^2}{2R}$$

$$\therefore \text{K.E.} = \frac{1}{4\pi \epsilon_0} \frac{e^2}{4R}$$

But $R = 2$ fm $= 2 \times 10^{-15}$ m,

$e = 1{\cdot}6 \times 10^{-19}$ C

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$$\therefore \text{K.E.} = \frac{9 \times 10^9 \times (1{\cdot}6 \times 10^{-19})^2}{4 \times 2 \times 10^{-15}}$$

$$= 2{\cdot}88 \times 10^{-14} \text{ J.}$$

$$\therefore \text{Coulomb barrier} = \frac{2{\cdot}88 \times 10^{-14}}{1{\cdot}6 \times 10^{-19}} eV$$

$= 180000\ eV =$ **180 keV**

13.21 From the relation R = R0A1/3, where R0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Ans - $R = R_0 A^{1/3}$

Where,

R_0 = Constant.

A = Mass number of the nucleus

$$\rho = \frac{M}{V} = \frac{Am}{\frac{4}{3}\pi R^3} \quad (m = \text{mass of a nucleon})$$

$$= \frac{Am}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Thus, nuclear density is independent of A.

13.22 For the β+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K–shell, is captured by the nucleus and a neutrino is emitted).
1 A A eX Y Z Z v + +→ + − Show that if β+ emission is energetically allowed, electron capture is necessarily allowed but not vice–versa.

Ans -
This means that if Q1 > 0, then Q2 > 0, but vice versa isn't always possible.

Therefore, positron emission can occur without electron capture.

For the positron emission, we have

$$ {}^{A}_{Z}X \rightarrow {}_{Z-1}^{A}X + {}^{0}_{+1}e + \nu + Q_1 \quad ...(1) $$

And for Electron Capture, we have

$$ {}^{A}_{Z}X + {}^{0}_{-1}e \longrightarrow {}_{Z-1}^{A}Y + \nu + Q_2 \quad ...(2) $$

If m_N represents the nuclear mass and m represents the atomic mass, then for m_e as the mass of ${}^{0}_{-1}e$ or ${}^{0}_{+1}e$, we have

Nuclear mass of ${}^{A}_{Z}X$

$$ = m_N\left({}^{A}_{Z}X\right) = m\left({}^{A}_{Z}X\right) - Z m_e $$

Nuclear mass of ${}_{Z-1}^{A}Y$

$$ = m_N\left({}_{Z-1}^{A}Y\right) = m\left({}^{A}_{Z}Y\right) - (Z-1) m_e $$

$$ \therefore Q_1 = \left[m_N\left({}^{A}_{Z}X\right) - m_N\left({}_{Z-1}^{A}Y\right) - m_e\right]c^2 $$

$$ = \left[m\left({}^{A}_{Z}X\right) - Zm_e - m\left({}_{Z-1}^{A}Y\right) + (Z-1)m_e - m_e\right]c^2 $$

$$ \Rightarrow Q_1 = \left[m\left({}^{A}_{Z}X\right) - m\left({}_{Z-1}^{A}Y\right) - 2m_e\right]c^2 $$

...(3)

13.23 In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are 24 12Mg (23.98504u), 25 12Mg (24.98584u) and 26 12Mg (25.98259u). The natural abundance of 24 12Mg is 78.99% by mass. Calculate the abundances of other two isotopes.

Ans -Average atomic mass of magnesium, m = 24.312 u

$$= \frac{78.99 \times 23.98504 + x \times 24.98584 + (100 - x - 78.99) \times 25.98259}{100}$$

$\Rightarrow 24.312 \times 100$

$$= 1894.58 + 24.98584x + 2598.259 - 25.98259x - 2052.36$$

or $25.98259x - 24.98584$

$= 1894.58 + 2598.259 - 2052.36 - 2431.2$

or $0.99675x = 9.279$

$$\therefore \quad x = \frac{9.279}{0.99675} = 9.31$$

$\therefore$ Natural abundance of ${}^{25}_{12}Mg$ is 9.31%

Also, natural abundance of ${}^{26}_{12}Mg$ is $(100 - 9.31 - 78.99)\% = 11.7\%$

13.24 The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei 41 20Ca and 27 13Al from the following data: m(40 20Ca) = 39.962591 u m(41 20Ca) = 40.962278 u m(26 13Al) = 25.986895 u m(27 13Al) = 26.981541 u

Ans - E = Energy equivalent of total mass afterward - Energy equivalent of nucleus before can be used to calculate the Ca's neutron separation.

For ${}^{41}_{20}Ca$, the process of neutron separation is

$${}^{41}_{20}Ca + E = {}^{40}_{20}Ca + {}^{1}_{0}n$$

$$\therefore E = \left[m\left({}^{40}_{20}Ca\right) + m_n - m\left({}^{41}_{20}Ca\right)\right] \times 931.5 \text{ MeV}$$

$= [39.962591 + 1.008665 - 40.962278] \times 931.5$ MeV

$= 0.008978 \times 931.5 = 8.36$ MeV

For ${}^{27}_{13}Al$, the process of neutron separation is

$${}^{27}_{13}Al + E = {}^{26}_{13}Al + {}^{1}_{0}n$$

$$\therefore E = \left[m\left({}^{26}_{13}Al\right) + m_n - m\left({}^{27}_{13}Al\right)\right] \times 931.5 \text{ MeV}$$

$= [25.986895 + 1.008665 - 26.981541] \times 931.5$ MeV

$= 0.014019 \times 931.5 = 13.06$ MeV.

13.25 A source contains two phosphorous radio nuclides 32 15P (T1/2 = 14.3d) and 33 15P(T1/2 = 25.3d). Initially, 10% of the decays come from 33 15 P. How long one must wait until 90% do so?

Ans - In the mixture of P-32 and P-33 initially 10% decay came from P-33. Hence initially 90% of the mixture is P-32 and 10% of the mixture is P-33. Let after time't' the mixture is left with 10% of P-32 and 90% of P-33. Half life of both P-32 and P-33 are given as 14.3 days and 25.3 days respectively. Let V be total mass undecayed initially and 'y' be total mass undecayed finally. Let initial number of P-32 nuclides = 0.9 x Final number of P-32 nuclides = 0.1 y Similarly, initial number of P-33 nuclides = 0.l x Final number of P-33 nuclides = 0.9 y For isotope P-32

Initially, source has 90% of ${}^{32}_{15}P$ and 10% of ${}^{33}_{15}P$. Let

after t days, source has 10% of ${}^{32}_{15}P$ and 90% of ${}^{33}_{15}P$.

$\therefore$ Initial number of ${}^{32}_{15}P = 9x$

Initial number of ${}^{33}_{15}P = x$

Final number of ${}^{32}_{15}P = y$

Final number of ${}^{33}_{15}P = 9y$

Now $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$\therefore \quad N = N_0(2)^{-t/T_{1/2}}$$

For first isotope,

$$y = 9x\,(2)^{-t/14\cdot3} \qquad \ldots(1)$$

For second isotope,

$$9y = xe^{-t/25\cdot3} \qquad \ldots(2)$$

Dividing (2) by (1)

$$9 = \frac{1}{9}(2)^{t\left(\frac{1}{14\cdot3}-\frac{1}{25\cdot3}\right)}$$

$$81 = 2^{\frac{11t}{14\cdot3\times25\cdot3}}$$

$$\log 81 = \frac{11t}{14\cdot3\times25\cdot3}\log 2$$

13.26 Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes: 223 209 14 88Ra Pb C 223 219 4 88Ra Rn He → + 86 2 Calculate the Q-values for these decays and determine that both are energetically allowed.

Ans - Mass of $^{223}_{88}\text{Ra}$ $m_1 = 223.01850$ u

Mass of $^{209}_{82}\text{Pb}$ $m_2 = 208.98107$ u

(*a*) The given decay process for $^{223}_{88}\text{Ra}$ is

$$^{223}_{88}\text{Ra} \rightarrow {}^{209}_{82}\text{Pb} + {}^{14}_{6}\text{C} + \text{Q}$$

$$\therefore \text{Q} = \left[m_{\text{N}}\left(^{223}_{88}\text{Ra}\right) - m_{\text{N}}\left(^{209}_{82}\text{Pb}\right) - m_{\text{N}}\left(^{14}_{6}\text{C}\right)\right] \times 931{\cdot}5\ \text{MeV}$$

where m_{N} denotes the nuclear mass. If m represents the atomic mass, then

$$\text{Q} = \left[m\left(^{223}_{88}\text{Ra}\right) - 88m_e - m\left(^{209}_{82}\text{Pb}\right) + 82m_e - m\left(^{14}_{6}\text{C}\right) + 6m_e\right] \times 931{\cdot}5\ \text{MeV}$$

$$= \left[m\left(^{223}_{88}\text{Ra}\right) - m\left(^{209}_{82}\text{Pb}\right) - m\left(^{14}_{6}\text{C}\right)\right] \times 931{\cdot}5\ \text{MeV}$$

$$= [223{\cdot}01850 - 208{\cdot}98107 - 14{\cdot}00324] \times 931{\cdot}5$$

$$= 31{\cdot}85\ \text{MeV}.$$

Also, another decay for $^{223}_{88}\text{Ra}$ is

$$^{223}_{88}\text{Ra} = {}^{219}_{86}\text{Rn} + {}^{4}_{2}\text{He} + \text{Q}'$$

$$\therefore \text{Q}' = \left[m_{\text{N}}\left(^{223}_{88}\text{Ra}\right) - m_{\text{N}}\left(^{219}_{86}\text{Rn}\right) - m_{\text{N}}\left(^{4}_{2}\text{He}\right)\right] \times 931{\cdot}5\ \text{MeV}$$

$$U = \frac{Z_1 Z_2 e^2}{4\pi \in_0 (r_1 + r_2)}$$

where Z_1 is the atomic no. of the particle and r_1 is its radius. Similarly Z_2 is the atomic number of daughter nucleus and r_2 is its radius.

But $r_1 = r_0 A_1^{1/3}$ and $r_2 = r_0 A_2^{1/3}$ where r_0 is nuclear unit radius.

$$\therefore \qquad U = \frac{Z_1 Z_2 e^2}{4\pi \in_0 r_0 [A_1^{1/3} + A_2^{1/3}]}$$

$\therefore$ For α– particle,

$$U(\alpha) = \frac{2 \times 86 \times (1{\cdot}6 \times 10^{-19})^2}{4\pi \in_0 r_0 [4^{1/3} + 219^{1/3}]}$$

$$= \frac{1}{4\pi \in_0 r_0} \times 5{\cdot}78 \times 10^{-37}$$

$$= \frac{5{\cdot}78 \times 10^{-37}}{4\pi \in_0 r_0} \text{ J}$$

For ${}^{14}_{6}C$, the barrier height

$$U\left({}^{14}_{6}C\right) = \frac{6 \times 82 \times (1{\cdot}6 \times 10^{-19})^2}{4\pi \in_0 r_0 [14^{1/3} + 209^{1/3}]}$$

$$= \frac{1{\cdot}51 \times 10^{-36}}{4\pi \in_0 r_0} \text{ J}$$

$$\therefore \frac{U\left({}^{14}_{6}C\right)}{U(\alpha)} = \frac{\dfrac{1{\cdot}51 \times 10^{-36}}{4\pi \in_0 r_0}}{\dfrac{5{\cdot}78 \times 10^{-37}}{4\pi \in_0 r_0}} = 2{\cdot}61$$

$$\therefore U\left({}^{14}_{6}C\right) = 2{\cdot}61 \times U(\alpha)$$

$$= 2{\cdot}61 \times 30 \text{ MeV}$$

$$(\because U(\alpha) = 30 \text{ MeV})$$

$$= 78{\cdot}3 \text{ MeV.}$$

13.27 Consider the fission of 238 92U by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are 140 58Ce and 99 44Ru. Calculate Q for this fission process. The relevant atomic and particle masses are m(238 92U) =238.05079 u m(140 58Ce) =139.90543 u m(99 44Ru) = 98.90594 u

Ans - Mass of a nucleus $\left({}^{238}_{92}\text{U}\right)$ m_1 = 238.05079 u

Mass of a nucleus $\left({}^{140}_{58}\text{Ce}\right)$ m_2 = 139.90543 u

Mass of a nucleus $\left({}^{99}_{44}\text{Ru}\right)$, m_3 = 98.90594 u

The fission process may be expressed as

$${}^{238}_{92}\text{U} + {}^{1}_{0}n \longrightarrow {}^{140}_{58}\text{Ce} + {}^{99}_{44}\text{Ru} + \text{Q}$$

$$\therefore \text{Q} = \left[m\left({}^{238}_{92}\text{U}\right) + m_n - m\left({}^{140}_{58}\text{Ce}\right) - m\left({}^{99}_{44}\text{Ru}\right)\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [238{\cdot}05079 + 1{\cdot}00867 - 139{\cdot}90543 - 98{\cdot}90594] \times 931{\cdot}5$$

$$= 231{\cdot}1 \text{ MeV.}$$

13.28 Consider the D–T reaction (deuterium–tritium fusion) 23 4 11 2 H H He n +→ +

(a) Calculate the energy released in MeV in this reaction from the data: m(2 1H)=2.014102 u m(3 1H) =3.016049 u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm.

What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic

energy available with the interacting particles = 2(3kT/2); k = Boltzman's constant, T = absolute temperature.)

Ans - Calculate the energy released in MeV in this reaction from the data:

$m\left({}_1^2\text{H}\right)$ = 2.014102 u

$$m\left({}_2^4\text{He}\right) + 2m_e - m_n\Big] \times 931{\cdot}5 \text{ MeV}$$

$$= \left[m\left({}_1^2\text{H}\right) + m\left({}_1^3\text{H}\right) - m\left({}_2^4\text{He}\right) - m_n\right] \times 931{\cdot}5 \text{ MeV}$$

$$= [2{\cdot}014102 + 3{\cdot}016049 - 4{\cdot}002603 - 1{\cdot}00867] \times 931{\cdot}5 \text{ MeV}$$

$$= 17{\cdot}585 \text{ MeV}.$$

(*b*) Repulsive potential energy is to be provided to the particles in doing so.

∴ K.E. needed

= Repulsive potential energy

$$= \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{2r} = \frac{1}{4\pi \epsilon_0} \frac{e^2}{2r}$$

$[q_1 = q_2 = e]$

$$= \frac{9 \times 10^9 \times (1{\cdot}6 \times 10^{-19})^2}{2(1{\cdot}5) \times 10^{-15}}$$

[distance between particle = $2r$]

$$= 7{\cdot}68 \times 10^{-14} \text{ J}$$

$$\text{K.E.} = \frac{3}{2}kT$$

$$T = \frac{2\text{K.E.}}{3k} = \frac{2 \times 7{\cdot}68 \times 10^{-14}}{3 \times 1{\cdot}38 \times 10^{-23}}$$

$$= 3{\cdot}7 \times 10^9 \text{ K}.$$

13.29 Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme

shown in Fig. 13.6. You are given that m(198Au) = 197.968233 u m(198Hg) =197.966760 u

Ans - Calculate the energy released in MeV in this reaction from the data:

$m\left({}_1^2H\right) = 2.014102$ u

$$\nu_1 = \frac{E_1}{h} = \frac{1.088 \times 1.6 \times 10^{-13}}{6.626 \times 10^{-34}}$$

$= 2.63 \times 10^{20}$ Hz

For γ_2, the frequency is

$$\nu_2 = \frac{E_2}{h} = \frac{0.412 \times 1.6 \times 10^{-13}}{6.626 \times 10^{-34}}$$

$= 9.95 \times 10^{19}$ Hz.

For γ_3, the energy $E_3 = 1.088 - 0.412$

$= 0.676$ MeV $= 0.676 \times 1.6 \times 10^{-13}$ J

$= 1.082 \times 10^{-13}$ J

$$\therefore \nu_3 = \frac{E_3}{h} = \frac{1.082 \times 10^{-13}}{6.626 \times 10^{-34}}$$

$= 1.63 \times 10^{20}$ Hz

Now for β_1^- decay,

$${}_{79}^{198}Au \longrightarrow {}_{80}^{198}Hg + {}_{-1}^{0}e + Q(\beta_1^-) + Q(\gamma_1)$$

Maximum kinetic energy

13.30 Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of 235U in a fission reactor.

Ans - (a) Amount of hydrogen, m = 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen $\left({}_1^1H\right)$ contains 6.023×10^{23} atoms.

(*a*) In sun fusion takes place according to the equation

$4\,{}^{1}_{1}H \longrightarrow {}^{4}_{2}He + 2\,{}^{0}_{+1}e + 26\text{ MeV}$

∴ 4 hydrogen atoms combine to produce 26 MeV of energy.

Now 1 g of hydrogen contains

$= 6{\cdot}02 \times 10^{23}$ nuclei

∴ 1000 g of hydrogen contains

$= 6{\cdot}02 \times 10^{23} \times 1000 = 6{\cdot}02 \times 10^{26}$ nuclei

∴ Energy released by 1 kg of hydrogen

$$= \frac{26\text{ MeV}}{4} \times 6{\cdot}02 \times 10^{26}$$

$= 3{\cdot}913 \times 10^{27}$ MeV

(*b*) Fission of one ${}^{235}_{92}U$ nucleus gives energy = 200 MeV.

13.31 Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of 235U to be about 200MeV.

Ans - Amount of electric power to be generated, $P = 2 \times 10^{5}$ MW

10% of this amount has to be obtained from nuclear power plants.

∴ Amount of nuclear power,

$$\text{Efficiency } \eta = \frac{\text{Total useful power}}{\text{Total power generated}}$$

∴ Total power generated

$$= \frac{\text{Total useful power}}{\eta}$$

$$= \frac{20^{10}}{\frac{25}{100}} = 8 \times 10^{10} \text{ W}$$

∴ Total energy required for the year 2020, is

$E = P \times t = 8 \times 10^{10} \times 366 \times 25 \times 60 \times 60$ (2020 is a leap year)

$= 2 \cdot 265 \times 10^{18}$ J.

Now 1 fission of $^{235}_{92}U$ produces 200 MeV of energy

$= 200 \times 1 \cdot 6 \times 10^{-13}$ J $= 3 \cdot 2 \times 10^{-11}$ J

∴ Number of fission required for the generation of energy E

$$= \frac{2 \cdot 265 \times 10^{18}}{3 \cdot 2 \times 10^{-11}} = 7 \cdot 90 \times 10^{28}.$$

Now $6 \cdot 023 \times 10^{23}$ nuclei of $^{235}_{92}U$ have mass = 235 g.

∴ Mass required to produce $7 \cdot 90 \times 10^{28}$ nuclei

$$= \frac{235}{6 \cdot 023 \times 10^{23}} \times 3 \cdot 95 \times 10^{28} \text{ g}$$

$\approx 3 \cdot 084 \times 10^{4}$ kg.