EXERCISE QUESTIONS

CHAPTER - 10 WAVE OPTICS

10.1 Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of
(a) reflected, and
(b) refracted light?
Refractive index of water is 1.33.

Ans - incident monochromatic light at a wavelength of 589 nm (589 x 10-9 m)
Light's velocity in air is 3 x 108 metres per second.
Water's refractive index is 1.33.
(i) The incident ray will be reflected back in the same medium that it travelled through initially.

The reflected beam will therefore have the same wavelength, speed, and frequency as the incident ray.

The relation: can be used to determine the frequency of light.

$$\therefore \text{ Frequency } \nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{589\times10^{-9}}$$
$$= 5{\cdot}09 \times 10^{14} \text{ Hz}$$

(*b*) *For refracted light,* while moving from one medium to the other, frequency remains unchanged. ∴ frequency of refracted light is $\nu' = 5{\cdot}09 \times 10^{14}$ Hz

$$\text{Wavelength, } \lambda' = \frac{\lambda}{n} = \frac{589}{1{\cdot}33} = 443 \text{ nm}$$

$$\text{and} \qquad v = \frac{c}{n} = \frac{3\times10^{8}}{1{\cdot}33}$$
$$= 2{\cdot}26 \times 10^{8} \text{ m s}^{-1}.$$

10.2 What is the shape of the wavefront in each of the following cases:
(a) Light diverging from a point source.

(b) Light emerging out of a convex lens when a point source is placed at its focus.
(c) The portion of the wavefront of light from a distant star intercepted by the Earth.

Ans - (A) When light is diverging from a point source, the wavefront has a spherical form.

(b) When a point source is put at the lens's focus, the wavefront of a light coming out of it takes the form of a parallel grid.

(c) The Earth-intercepted component of the wavefront of light from a far-off star is a plane**.**

10.3 (a) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3.0 × 108 m s–1)
(b) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism?

Ans - (a) Speed of light in glass,
Refractive index of glass, μ = 1.5

Speed of light, c = 3×10^8 m/s

$$v = \frac{c}{{}^a\mu_g} = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m s}^{-1}$$

(b) The colour of the light (i.e.,) affects how fast light travels through glass. As a result, the speed of light differs for the colours violet and red. Since v > R, v R follows, and v R Red colour light moves through glass more quickly than violet colour light.

10.4 In a Young’s double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

Ans - The distance between the slits is d = 0.28 mm = 0.28 x10-3 m.

D = 1.4 m is the distance between the slits and the screen.

the distance between the fourth (n = 4) fringe and the centre fringe

Here, $d = 0{\cdot}28 \text{ mm} = 0{\cdot}28 \times 10^{-3} \text{ m}$

$D = 1{\cdot}4 \text{ m}, \ m = 4$ and $y_m = 1{\cdot}2 \text{ cm}$

$= 1{\cdot}2 \times 10^{-2} \text{ m}$

$\therefore$ From the relation, $y_m = \dfrac{m\lambda D}{d}$,

$$\lambda = \frac{y_m d}{mD} = 1{\cdot}2 \times 10^{-2} \times \frac{0{\cdot}28 \times 10^{-3}}{4 \times 1{\cdot}4}$$

$$= 6 \times 10^{-7} \text{ m} = 600 \text{ nm}.$$

10.5 In Young's double-slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen where path difference is λ, is K units. What is the intensity of light at a point where path difference is λ/3?

Ans - Let I1 and I2 represent the two light waves' intensities. You can determine their resultant intensities as:

$$I = 4 I_0 \cos^2\left(\frac{\phi}{2}\right)$$

When, path diff. = λ, phase diff., $\phi = \dfrac{2\pi}{\lambda} \times \lambda = 2\pi$

$\therefore \ I = 4I_0 (\cos \pi)^2 = 4I_0 = K$ ...(*i*)

When, path diff. = $\dfrac{\lambda}{3}$, phase diff., $\phi = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{3} = \dfrac{2\pi}{3}$

$\therefore \ I' = 4I_0 \left[\cos\left(\dfrac{\pi}{3}\right)\right]^2 = 4I_0 \times \dfrac{1}{4} = \dfrac{K}{4}$ [Using eqn. (*i*)]

10.6 A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes in a Young's double-slit experiment.

(a) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.

(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?

Ans - Slits' separation from the screen = D

d is the separation between the two slits.

(a) The relation states how far the nth brilliant fringe on the screen is from the central maximum.

$$y_m = \frac{m\lambda D}{d}$$

$$\text{For } m = 3,\ y_3 = \frac{3 \times 650 \times 10^{-9} \times 1{\cdot}2}{2 \times 10^{-3}}$$

$$= 1{\cdot}17 \times 10^{-3}\ \text{m}$$

$$= 1{\cdot}17\ \text{mm}$$

(*b*) Let y' = Minimum distance of the position from the central maximum where mth bright fringe due to λ_1 and $(m + 1)$th bright fringe due to λ_2 coincide.

$$\text{Now} \quad y' = \frac{mD\lambda_1}{d} = \frac{(m+1)D\lambda_2}{d}$$

$$i.e. \ \frac{m}{m+1} = \frac{\lambda_2}{\lambda_1} = \frac{520}{650} = 0{\cdot}8$$

$$\text{or} \quad m = 0{\cdot}8 + 0{\cdot}8m \quad \text{or} \quad m = 4$$

$$\therefore \quad y' = m\left(\frac{D\lambda_1}{d}\right)$$

$$= \frac{4 \times 1{\cdot}2 \times 650 \times 10^{-9}}{2 \times 10^{-3}}$$

$$= 1{\cdot}56 \times 10^{-3}\ \text{m} = 1{\cdot}56\ \text{mm.}$$

10.7 In a double-slit experiment the angular width of a fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.

Ans - screen placed distance =1 m
Wavelength of light = 600nm

Angular width,

$$\theta = \frac{\lambda}{d} = 0{\cdot}2° \qquad ...(i)$$

When apparatus is immersed in water, wavelength

$$\lambda' = \frac{\lambda}{n}$$

∴ Angular width,

$$\theta' = \frac{\lambda'}{d} = \frac{\lambda}{nd} = \frac{\theta}{n} = \frac{0{\cdot}2° \times 3}{4} = 0{\cdot}15°.$$

10.8 What is the Brewster angle for air to glass transition? (Refractive index of glass = 1.5.)

Ans - $\mu = 1.5$
Brewster angle = θ

Brewster angle is related to refractive index as:
According to Brewster's law,

$\mu = \tan p$
$\tan p = 1.5$
$\Rightarrow p = 56.31^o$

10.9 Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

Ans - The wavelength and frequency of the reflected light are identical to those of the incident light. The wavelength of the reflected light is 5000 A, and its frequency is

Speed of light, $c = 3 \times 10^8$ m

Frequency of incident light is given by the relation, Also, $\nu = \frac{c}{\lambda} = \frac{3\times10^8}{5000\times10^{-10}} = 6 \times 10^{14}$ Hz

Now $i = r$

Also, $i + r = 90°$ (given)

$\therefore$ $i + i = 90°$ or $2i = 90°$ or $i = 45°$

10.10 Estimate the distance for which ray optics is good approximation for an aperture of 4 mm and wavelength 400 nm.

Ans - Fresnel distance necessary for the core bright fringe to extend sufficiently, allowing for detectable diffraction

$$Z_f = \frac{a^2}{\lambda}$$

$$= \frac{(4\times10^{-3})^2}{400\times10^{-9}} = 40 \text{ m.}$$

Ray optics is a decent approximation for distances less than 40 m between the slit and the screen since the spreading is minimal at these short distances.

10.11 The 6563 Å Hα line emitted by hydrogen in a star is found to be redshifted by 15 Å. Estimate the speed with which the star is receding from the Earth.

Ans - The star's velocity, v, is negative since it is moving away (i.e. if AA is positive, v is negative)

$\lambda = 6563$ Å

$\Delta\lambda = 15$ Å

Now using the relation

$$\Delta\lambda = -\frac{\upsilon}{c}\lambda \quad \text{(for red shift)}$$

$$\Rightarrow \upsilon = -\frac{\Delta\lambda}{\lambda}\cdot c$$

$$= -\frac{15}{6563} \times 3 \times 10^8$$

$$= -\frac{15}{6563} \times 3 \times 10^8 \text{ ms}^{-1}$$

$$= -6{\cdot}86 \times 10^5 \text{ m s}^{-1}.$$

The declining star can be seen in this negative sign.

10.12 Explain how Corpuscular theory predicts the speed of light in a medium, say, water, to be greater than the speed of light in vacuum. Is the prediction confirmed by experimental determination of the speed of light in water? If not, which alternative picture of light is consistent with experiment?

Ans - According to corpuscular theory, a force of attraction acts on the particles normal to the surface when light in the form of particles enters a denser media from a rarer medium. As a result, the component of water's surface velocity that is normal to the surface increases while the component that is parallel to the surface remains same. Therefore, Considering a light beam moving from a rarer (air) to a denser (matrix) medium (water). Let I = angle of incidence, r = angle of refraction, v = speed of light in water, c = speed of light in vacuum (or air), and Following that, based on Newton's corpuscular hypothesis,

$$c \sin i = v \sin r \quad \text{or} \quad \frac{v}{c} = \frac{\sin i}{\sin r} = n.$$

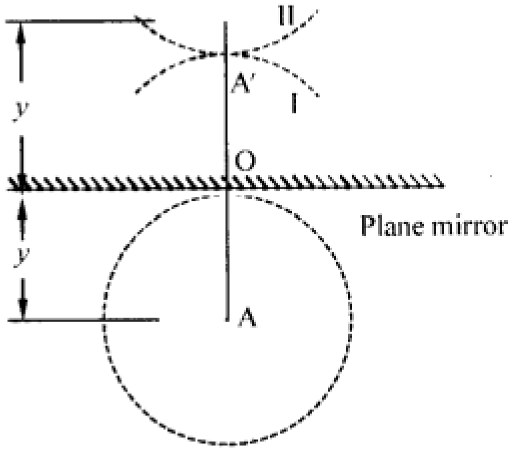
$$\text{But} \quad n > 1 \quad \therefore \quad \frac{v}{c} > 1 \quad \text{or} \quad v > c$$

The formula is: Component of velocity c along the separation surface equals Component of velocity v along the separation surface.

Light travels at a faster rate in water than in air. However, in reality, c >. The experiment is consistent with the light wave theory proposed by Huygens.

10.13 You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction. Use the same principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the object distance from the mirror.

Ans - P is a point object in the figure that is placed at a distance. r from a mirror in the air M1M2. Draw a spherical arc with P as the centre and PO = r as the radius; AB. This is the object's spherical wave front that was incident on M1M2. The location of wave front AB would be A'B' in the absence of mirrors, where PP' = 2r. According to Huygen's theory, wave front A B would look as A"PB" in the presence of the mirror. A'B' and A"B" are two spherical arcs that are equally situated on either side of M1M2, as is seen from the illustration. A'P'B' can be thought of as the reflected image of A"PB. We determine OP = OP', which needed to be proven, using elementary geometry.



10.14 Let us list some of the factors, which could possibly influence the speed of wave propagation:
(i) nature of the source.
(ii) direction of propagation.
(iii) motion of the source and/or observer.
(iv) wavelength.
(v) intensity of the wave. On which of these factors, if any, does
(a) the speed of light in vacuum,
(b) the speed of light in a medium (say, glass or water), depend?

Ans - (a) The speed of light in a vacuum is unaffected by any of the aforementioned variables. Additionally, it is unaffected by the mobility of the source in relation to the observer.
(b) The relationship between light speed and a medium.

(i) The kind of the source has no bearing on the speed of light in a given medium. Nevertheless, the characteristics of the propagation medium determine speed.
(ii) For an isotropic medium, the speed of light is unaffected by the direction of propagation.

(iii) The velocity of the source in relation to the medium has no bearing on the speed of light, whereas the motion of the observer in relation to the medium does.

(iv) The wavelength of light affects how fast light travels through a medium.

(v) The speed of light in a medium is independent of intensity

10.15 For sound waves, the Doppler formula for frequency shift differs slightly between the two situations:
(i) source at rest; observer moving, and
(ii) source moving; observer at rest. The exact Doppler formulas for the case of light waves in vacuum are, however, strictly identical for these situations. Explain why this should be so. Would you expect the formulas to be strictly identical for the two situations in case of light travelling in a medium?

Ans - For sound to travel, a physical medium is needed. Situations (1) and (2) may both involve the same relative motion, but they are not physically identical since the motion of the observer in relation to the medium differs in each case. Doppler effects for sound cannot therefore be the same in both scenarios. Different Doppler equations for different types of materials regulate how light behaves when passing through them.

Both a source at rest and an observer in motion can occur at the same time.
However, since the speed of light and its frequency/wavelength are unaffected by vacuum, the formulas for the two separate conditions are exactly the same when light travels through it.

10.16 In double-slit experiment using light of wavelength 600 nm, the angular width of a fringe formed on a distant screen is 0.1°. What is the spacing between the two slits?

Ans -

Wavelength = 600nm

Angular width =0.1

$$\text{Angular width, } \theta = \frac{\lambda}{d} \text{ or } d = \frac{\lambda}{\theta}$$

$$\text{Now, } \lambda = 600 \text{ nm}$$

$$= 600 \times 10^{-9}\text{m} = 6 \times 10^{-7} \text{ m}$$

$$\theta = 0.1° = \frac{0.1}{180} \times \pi \text{ rad}$$

$$\therefore \quad d = \frac{6 \times 10^{-7}}{0.1 \times \pi} \times 180 = 3.5 \times 10^{-4}\text{m}$$

10.17 Answer the following questions:
(a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band?
(b) In what way is diffraction from each slit related to the interference pattern in a double-slit experiment?
(c) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the shadow of the obstacle. Explain why?
(d) Two students are separated by a 7 m partition wall in a room 10 m high. If both light and sound waves can bend around obstacles, how is it that the students are unable to see each other even though they can converse easily.
(e) Ray optics is based on the assumption that light travels in a straight line. Diffraction effects (observed when light propagates through small apertures/slits or around small obstacles) disprove this assumption. Yet the ray optics assumption is so commonly used in understanding location and several other properties of images in optical instruments. What is the justification?

Ans - (A) The centre maxima's breadth is equal to 2D/d.
When the slit's width (d) is doubled, the width of the central diffraction maxima is cut in half, and as the amplitude of the light wave doubles, the intensity of the central band increases four times.

(b) The superposition of the diffraction patterns caused by each slit causes the intensity of the fringes generated in the double-slit experiment to fluctuate.

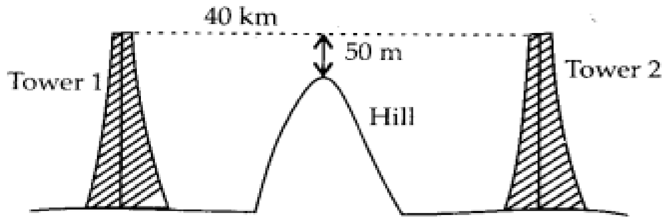
(c) The edge of the circular obstruction diffracts waves from a distant source, and these waves then constructively superimpose at the centre of the obstacle's shadow to produce a bright spot.

(d) We are aware that the size of the obstruction or aperture must be on the order of the wavelength for diffraction to occur. Sound waves can bend through a large opening in a partition wall because their wavelength is on the order of a few metres, while light waves cannot because their wavelength is on the order of a micrometre. Due to this, although being unable to see one another, the two pupils can hear one another.

(e) In optical instruments, size of apertures are much larger than the wavelength of light. So diffraction of light is negligible. Hence, the assumption that light can travel in straight line is used in optical instruments.

10.18 Two towers on top of two hills are 40 km apart. The line joining them passes 50 m above a hill halfway between the towers. What is the longest wavelength of radio waves, which can be sent between the towers without appreciable diffraction effects?

Ans - :



For diffraction of radio waves not to occur the distance of middle hill should be less than fresnel distance for a slit width ‘a’ of 50 m.

Here, $D = \frac{40}{2} = 20$ km

$= 2 \times 10^4$ m

Fresnel distance, Z_F must be less than 50 m.

Since $Z_F = \sqrt{\lambda D}$

$\therefore \sqrt{\lambda D} < Z_F$

or $\lambda D < Z_F^2$

or $\lambda < \frac{Z_F^2}{D} < \frac{2500}{2 \times 10^4}$

$= \mathbf{12 \cdot 5}$ **cm.**

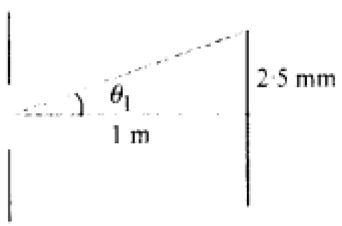
10.19 A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit. 10.20 Answer the following questions:
(a) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.
(b) As you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns. What is the justification of this principle?

Ans - The first minimum is noticed 2.5 mm from the screen's centre.
Resulting diffraction observed = 1m

$$\text{For first minimum } \sin\theta_1 = \frac{\lambda}{d}$$

$$\text{But } \sin\theta_1 = \tan\theta_1 = \frac{2{\cdot}5\times10^{-3}}{1}$$

$$= 2{\cdot}5\times10^{-3}$$



$$\therefore d = \frac{\lambda}{\sin\theta_1} = \frac{500\times10^{-9}}{2{\cdot}5\times10^{-3}}$$

$$= 0{\cdot}2 \text{ mm.}$$

10.21 In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles of nλ/a. Justify this by suitably dividing the slit to bring out the cancellation.

Ans - (a) The metallic aircraft body reflects the TV signal when a low-flying aircraft passes overhead. Due to interference between the direct signal received by the antenna and the direct signal from the aeroplane, there is a minor trembling of the image on the TV screen.

(b) Another wave equation is the linear combination of wave equations. The superposition principle is based on this in its purest form.